CALCULUS.

133. Proposed by NELSON L. RORAY, South Jersey Institute, Bridgeton, N. J.

Integrate $\int \frac{\sqrt{[1+y]}}{1+y^2} dy$.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics in The Temple College, Philadelphia, Pa.

Let $1+y=z^2$.

$$\therefore \int \frac{\sqrt{[1+y]}}{1+y^2} dy = \int \frac{2z^2 dz}{1+[z^2-1]^2} = [1+\sqrt{-1}] \int \frac{dz}{1-\sqrt{-1}+z^2\sqrt{-1}}$$

$$+[1-\sqrt{-1}] \int \frac{dz}{1+\sqrt{-1}-z^2\sqrt{-1}} = -[1-\sqrt{-1}] \int \frac{dz}{1+\sqrt{-1}-z^2}$$

$$-[1+\sqrt{-1}] \int \frac{dz}{1-\sqrt{-1}-z^2} = u.$$

Let $1+\sqrt{-1}=a^2$, $1-\sqrt{-1}=b^2$.

$$\therefore u = -\frac{1}{a^3} \int \left[\frac{1}{a+z} + \frac{1}{a-z}\right] dz - \frac{1}{b^3} \int \left[\frac{1}{b+z} + \frac{1}{b-z}\right] dz$$

$$= \frac{1}{a^3} \log \left[\frac{a-z}{a+z}\right] + \frac{1}{b^3} \log \left[\frac{b-z}{b+z}\right] + C.$$

$$\therefore u = \left[\frac{1}{1+\sqrt{-1}}\right]^{\frac{3}{2}} \log \left[\frac{\sqrt{[1+\sqrt{-1}]} - \sqrt{[1+y]}}{\sqrt{[1+\sqrt{-1}]} + \sqrt{[1+y]}}\right]$$

$$+ \left[\frac{1}{1-\sqrt{-1}}\right]^{\frac{3}{2}} \log \left[\frac{\sqrt{[1-\sqrt{-1}]} - \sqrt{[1+y]}}{\sqrt{[1-\sqrt{-1}]} + \sqrt{[1+y]}}\right] + C.$$

Also solved by *F. P. MATZ*. An incorrect solution was received from *H. C. WHITAKER*.

134. Proposed by J. SCHEFFER, A. M., Hagerstown, Md.

To find the curve for which the sum of that part of the tangent, lying between the point of contact and the axis of abscissas, and the corresponding ordinate is constant$=c$, and which passes through the point (a, b).

Solution by F. P. MATZ, Sc. D., Ph. D., Defiance College, Defiance, O.; G. B. M. ZERR, A. M., Ph. D., The Temple College, Philadelphia, Pa.; COOPER D. SCHMIDT, A. M., University of Tennessee, Knoxville, Tenn.; and the PROPOSER.

According to the conditions of the problem, *Ordinate*$+$*Tangent*$=$*Constant*.

That is, $y+y\sqrt{\left(\frac{dx}{dy}\right)^2+1}=C$....[1].

Therefore, $dx=\dfrac{\sqrt{[C^2-2Cy]}}{y}dy$....[2].

or, $x=2\sqrt{[C^2-2Cy]}+C\log\left[\dfrac{\sqrt{[C^2-2Cy]}-C}{\sqrt{[C^2-2Cy]}+C}\right]+C'$....[3].

For $x=a$ and $y=b$, as per the problem, [3] gives the required equation

$$x-a=2\{\sqrt{[C^2-2Cy]}-\sqrt{[C^2-2bC]}\}$$

$$+C\log\left[\frac{\sqrt{[C^2-2Cy]}-C}{\sqrt{[C^2-2Cy]}+C}\Big/\frac{\sqrt{[C^2-2bC]}-C}{\sqrt{[C^2-2bC]}+C}\right]....[4].$$

Also solved by *L. C. WALKER* and *H. C. WHITAKER*. Professor Matz gave a second solution using polar co-ordinates.

135. Proposed by COOPER D. SCHMITT, A. M., Professor of Mathematics, University of Tennessee, Knoxville, Tenn.

To find the equation of the evolute of the common catenary

$$y=(\tfrac{1}{2}c)(e^{x/c}+e^{-x/c}).$$

Solution by G. B. M. ZERR, A. M., Ph. D., The Temple College, Philadelphia, Pa., and M. E. GRABER, Heidelberg University, Tiffin, O.

$$\frac{dy}{dx}=\tfrac{1}{2}(e^{x/c}-e^{-x/c}),\quad \frac{d^2y}{dx^2}=\frac{1}{2c}(e^{x/c}+e^{-x/c}).$$

$\therefore$ $x-m+(y-n)dy/dx=0$, $1+(dy/dx)^2+(y-n)d^2y/dx^2=0$ become

$$x-m+\tfrac{1}{4}c(e^{2x/c}-e^{-2x/c})-\tfrac{1}{2}n(e^{x/c}-e^{-x/c})=0....(1).$$

$$1+\tfrac{1}{4}(e^{x/c}-e^{-x/c})^2+\tfrac{1}{4}(e^{x/c}+e^{-x/c})^2-\frac{n}{2c}(e^{x/c}+e^{-x/c})=0....(2).$$

From (2), $e^{4x/c}-\dfrac{n}{c}e^{3x/c}+2e^{2x/c}-\dfrac{n}{c}e^{x/c}+1=0$.

Let $e^{x/c}=z$.

$\therefore$ $z^4-\dfrac{n}{c}z^3+2z^2-\dfrac{n}{c}z+1=0$, $(z^2+1)(z^2-\dfrac{n}{c}z+1)=0$.

$\therefore$ $z=\dfrac{n\pm\sqrt{(n^2-4c^2)}}{2c}=e^{x/c}$ and $\dfrac{n\mp\sqrt{(n^2-4c^2)}}{2c}=e^{-x/c}$.

$\therefore$ $x=c\log\left(\dfrac{n\pm\sqrt{(n^2-4c^2)}}{2c}\right)$.